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A Learning Community for Pre-Service Elementary Teachers: A Collaboration Between Mathematics and Education

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Abstract: This article describes a collaboration between two faculty members, one from mathematics and the other from education, to strengthen the preparation of pre-service elementary school teachers for teaching mathematics. The model used is a learning community which combines a mathematics content course offered by the mathematics department, with a mathematics teaching methods course taught by the education department. This article details the design features of the learning community and its rationale and administrative implementation. Course materials and grading policy are discussed, as well as methods of assessment and interpretation of assessment. Of special note, this paper reports the benefits of the learning community as perceived by students and faculty.

Keywords: Learning community, Mathematical Knowledge for Teaching, NCTM Principles and Standards.

1. INTRODUCTION

For over a decade, K-12 students in the United States have been faltering on national, as well as international, normed mathematics exams.

> "Despite a widely held belief that U.S. students do well in mathematics in grade school but decline precipitously in high school, a new study comparing the math skills of students in industrialized nations finds that U.S. students in 4th and 8th grade perform consistently below most of their peers around the world and continue that trend into high school" [13].

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On the 2005 National Assessment of Educational Progress in mathematics, " . . . most students did not reach the proficient performance level, a level denoting solid performance for their grade based on judgments of what students should know and be able to do in the subject assessed" [16]. Only about one-third of 4th and 8th grade students, and less than one-fourth of 12th grade students, reached the proficient level [19].

Researchers in mathematics education who have been comparing teaching practices in the United States with those in high-performing nations, conclude:

> "Although the typical methods of improving U.S. instructional quality have been to develop curriculum, and—especially in the last decade—to articulate standards for what students should learn, little improvement is possible without direct attention to the practice of teaching" [2, p. 14].

> "Many U.S. teachers lack sound mathematical knowledge and skill" [2, p. 14]. Their understanding of mathematics and mathematics pedagogy is "thin"[4, p. 17].

> "The fact that their understanding is more rule-bound than conceptual, and more fragmented than connected, reflects the nature of the teaching and curriculum that they, like other American adults, experienced in elementary and secondary schools" [4, p. 18].

To improve the teaching and learning of mathematics at all grade levels in the United States, mathematics and mathematics education professional societies are urging faculty to collaborate in order to enhance the preparation of teachers. In its 2001 report, *The Mathematical Preparation of Teachers*, the Conference Board of the Mathematical Sciences recommended: "The mathematical education of teachers should be seen as a partnership between mathematics faculty and mathematics education faculty"[9, Recommendation 6]. The final report from a 2005 workshop for mathematicians and educators, Mathematical Knowledge for Teaching (K-8): Why, What, and How?, sponsored by the Mathematical Sciences Research Institute, called on mathematicians to contribute their expertise to help identify the mathematical knowledge teachers need for teaching, and to ". . . seek out collaborations with (education) colleagues" [18, p. 19].

Responding to these calls, we created a collaboration between two faculty members, one from mathematics and the other from education, to strengthen and improve the preparation of pre-service elementary school teachers for teaching mathematics. The model we developed and used is a learning community. The audience for this article is mathematics faculty who may wish to undertake the learning community model themselves. We describe its key features, and the unique contributions to our work that proceeded from our

participation in two institutes of the Center for Proficiency in Teaching Mathematics (CPTM) at the University of Michigan-Ann Arbor. We report on the benefits of the collaboration from both the faculty and student perspectives. We conclude with lessons learned and future directions.

2. THE LEARNING COMMUNITY MODEL

2.1. Background to the Initiative

The authors teach at a private, comprehensive university in Boston. At our school, like many others, the mathematics and education departments share joint, but independent, responsibility for teacher preparation; the mathematics department offers a content course for pre-service elementary school teachers, Mathematics for Elementary School Teachers, and the education department offers a mathematics methods course, Mathematics in the Elementary and Early Childhood Classroom. The authors are the instructors who developed and teach those courses.

On meeting at a mathematics education conference in fall 2003, we found that we shared the constructivist philosophy of learning and favor active learning strategies. Above all, we perceived the need to strengthen and improve teacher preparation for teaching elementary school mathematics. An opportunity for formal collaboration emerged in December 2004 when the Dean of the College of Arts and Sciences announced a college-wide learning community initiative. We proposed a learning community for pre-service elementary school teachers that would integrate learning the content of elementary school mathematics with learning best teaching practices. The learning community was launched in spring 2006. It has been offered each spring semester since.

2.2. Design Features

Learning communities generally share three features: They target a specific cohort of students; they link two or more courses [22]; and they include an integrative seminar. Our learning community has all three features: It targets undergraduates who aspire to become elementary school teachers; it links the mathematics department's content course with the education department's mathematics teaching methods course; and it has a weekly, integrative seminar which is co-led by the mathematics and education instructors.

2.3. Rationale

At the first meeting of the integrative seminar, students receive a handout in which the learning goals for the learning community are articulated. The

overall goal of our learning community is to help pre-service elementary school teachers build professional competence and personal confidence in a central domain of teaching: elementary school mathematics. This learning goal is formalized as follows:

> To integrate learning elementary school mathematics with learning to teach this content confidently and effectively, measured by your knowledge, as an adult, of the concepts and procedures used in elementary school mathematics, and by your skill in making this knowledge accessible to children (e.g., through your use of language, and your choice and use of models or representations—verbal, visual, symbolic, numerical).

For context, we note that prior to the learning community initiative, elementary education students often took the mathematics content and pedagogy courses in different semesters, even in different years. A practical reason for linking the courses was the desire to be more efficient; having students study the concepts and procedures of elementary school mathematics as they are learning how to teach them, reduces the time needed for reviewing the mathematics in the methods course.

Beyond efficiency, the compelling reason for linking content and pedagogy was to address the problem of thin understanding of mathematics. For many American adults, not just pre-service teachers, mathematics consists of memorized rules, formulas, and procedures; as Keith Devlin notes in his article, *What Is Conceptual Understanding?*, ". . . procedural skills that are not eventually accompanied by some form of understanding are brittle and easily lost"[10].

The mathematics learning community at Simmons College offers students the opportunity and the responsibility to solidify their understanding of the concepts of elementary school mathematics. As students re-trace the familiar landscape of elementary school mathematics, they are continually asked: Why does this algorithm work? What are the mathematical principles behind this rule? So that they will be able to help children learn mathematics with understanding, we require our students to reflect on their individual learning trajectories as a tool for thinking about how learning mathematics takes place. Deep, conceptual understanding is another explicit learning goal of the learning community:

> To promote deep understanding of mathematics beyond mere rote memorization of facts and procedure, particularly understanding of place value, number systems and operations, and fractions (different meanings and representations), as measured by your ability to use a variety of models (visual, symbolic, physical) to represent these concepts and processes and to draw connections between the models.

2.4. Role and Format of the Integrative Seminar

The weekly integrative seminar plays a pivotal role in the learning community: helping students make the transition from knowing and understanding the concepts, rules, and procedures of elementary school mathematics for themselves, to being able to make this content comprehensible and accessible to children. The integrative seminar is co-taught by the mathematics and education instructors. We practice two forms of collaborative teaching: *alternate teaching*, where one faculty member runs the entire session and the other plays a support role [8]; and *team-teaching*, where we co-present, respond to each other's presentation and co-facilitate student discussion [8]. We use a variety of *bridging practices,* e.g., observation and discussion of both videotaped and in-person clinical interviews of children; observation and discussion of videotapes of classroom instruction [1]; and analysis of pieces of mathematical work done by children.

In the first year of the learning community, when we were getting to know each other and each other's disciplinary content and teaching styles, we mostly practiced alternate teaching. Both instructors actively participated in every session although we divided the leadership evenly. We always co-planned and shared decisions about the content and organization of each session, but we mostly taught individually.

When the mathematics instructor led the integrative seminar, she focused on a specific mathematics topic, and the education instructor followed by relating how this content plays out in the elementary school mathematics curriculum. For example, in one session, where the mathematician covered modeling changes over time with graphs of functions, the education faculty member showed how this topic gets developed in current elementary school curricula and also had students review related questions from recent high-stakes, mathematics exams given to elementary school children in Massachusetts.

When the education instructor led the session, she often presented a common mathematics activity or problem, and the mathematics instructor followed up by delving further into the mathematics. For example, in one session, the education professor presented the *King's Chessboard,* a problem which crosses many different curricula and for which many teachers lack a firm grounding in the mathematics. The purpose of the education professor's presentation was to use this problem to help students understand the nature of exponential growth and exponential notation. The mathematics instructor then led students to apply exponential notation to develop the sums-of-powers-of-two formula,

$$1 + 2 + 2^2 + 2^3 + \ldots + 2^N = 2^{N+1} - 1.$$

As we gained experience with collaborative teaching, we were able to practice team-teaching. For example, we team-taught a session around the *Valentine Exchange* problem:

> We have 24 students in our class. If each student gives a Valentine to everyone in the class, how many Valentines will there be? [20]

This problem appears in various forms both in mathematics textbooks for preservice elementary school teachers and in elementary school curricula. During the integrative seminar, 20 minutes were spent viewing the video, *Valentine Exchange* [20], from the Annenberg Math K-4 video library. This gave students the opportunity to observe a fourth grade teacher carry out a lesson devoted to this problem. The next 20 minutes were spent discussing the teacher's moves, e.g., her responses to student claims and her questions to probe children's understanding of the problem and the thinking behind their strategies.

The remainder of class time was spent discussing the fourth grade students' mathematical ideas and the mathematics that our students would need to know well in order to teach this lesson effectively. For example, students considered whether one child's approach of solving a smaller problem first, and then scaling up, was reasonable. They also were asked to identify the various representations children had used to solve the problem (e.g., making a list of all the Valentine exchanges, making a grid, finding an arithmetic formula), and to discuss the connections between them.

The integrative seminar is an essential component of the learning community. Its importance is reflected in our grading policies, to be described later in this article.

2.5. Culture

A vital, though intangible, feature of the learning community is the sense of purpose which is shared by students and faculty. All members are committed to the profession of teaching; also, all members are learners: The students are learning the content of elementary school mathematics and best ways to teach it, and the instructors are gaining knowledge and appreciation of each other's discipline and expertise. An added value to future teachers is that the instructors are modeling collaboration.

In the first year, the instructors attended all of each other's classes and also co-taught the integrative seminar. This helped to sustain the link between the content and pedagogy courses and to promote the students' continuous engagement with the question: What is the difference between knowing elementary school mathematics yourself, and knowing mathematics well enough to be able to teach elementary school mathematics to children? The following learning goal captures the learning environment we strive to create:

> To support each other as fellow learners and teachers, and to promote intellectual engagement and interaction: student with student, student with faculty, and faculty with faculty, as measured by the quality and

quantity of your in-class participation (e.g., listening and responding to the mathematical claims of others, sharing your solutions, as well as your confusions and questions).

At the end of the semester, when students were asked, "What aspect(s) of the learning community were strengths?," they pointed to the culture of the learning community.

Here are some representative student responses to this question:

- "Strengths: small class size, strong culture/community, we saw each other *frequently*, worked together for most things, explained to our classmates, so it was easy to ask questions or admit confusion, this never seemed a problem/embarrassment. Everyone talked which is really amazing."
- "Strengths were the relationships with other students, the small lively classes, and the math itself."
- "I liked learning the math and ways of teaching together. It made more sense overall and I could connect to the math better."

3. IMPLEMENTATION OF THE LEARNING COMMUNITY

We have described the overall design features of our mathematics learning community and we now turn to the specifics of its implementation.

3.1. Administrative Details

There are fundamental administrative considerations for implementing a learning community. These include registration, course credits, and faculty workload. The formal association of the mathematics content and mathematics methods courses for pre-service teachers is effected at registration; students who register for one course are automatically registered for the other.

In terms of credit hours, non-laboratory courses at our school typically meet three hours per week and carry four credits. For the mathematics learning community, students meet six hours per week: two hours each in the mathematics content course, the mathematics methods course, and the integrative seminar. They also are required to observe actual elementary classrooms twice every week. Students receive separate grades for the content and methods courses and earn 8 credits (the equivalent of two courses). The integrative seminar is weighted 25 percent in each course, reflecting its central role in the learning community. The grading policies for the mathematics content and methods courses will be presented shortly.

Faculty workload at our school is typically three courses per semester. The year that we launched the learning community, we each received one

course release; this time was used for planning, preparation, and attending all class meetings of each other's course.

Similar to freshmen writing courses and honors seminars at many institutions, the class size for the learning community has been roughly a dozen. As noted earlier, students regard small class size as a strength; it promotes lively class discussion and a learning environment that supports and engages all students.

3.2. Structure of the Mathematics Content Course

The focus of the mathematics content course is conceptual understanding of elementary school mathematics. The topics of the course are problem-solving, number systems and operations, fractions, sequences and functions, elementary data analysis, and calculating probabilities. For the mathematics content course, we selected Sybilla Beckmann's textbook, *Mathematics for Elementary Teachers* [6], with its accompanying activities manual [7], because it supports our learning goal of deep understanding:

> "I wrote this book to help future elementary school teachers develop a deep understanding of the mathematics that they will teach. . . . [T]o teach mathematics well, teachers must know more than just how to carry out basic mathematics procedures; they must be able to explain why mathematics works the way it does. . . . This book focuses on 'explaining why'" [6, p. ix].

Course assignments consist of daily homework from [6], as well as exploratory activities from its companion activities manual, [7]. These assignments are intended to develop students' facility in: using mathematical terminology, notation, and formulas; writing to explain their thinking; drawing mathematical diagrams (e.g., graphs of functions or pie charts or histograms); and, performing multi-step numerical calculations and algorithms. As we noted earlier, the integrative seminar is intended to help students make the transition from learner to teacher, and to anticipate how children, who will be seeing the mathematical content for the first time, build their understanding.

The course point distribution for the mathematics content course appears in Table 1.

3.3. Structure of the Mathematics Methods Course

The teaching methods course is based on the constructivist philosophy of learning. The focus of the course is on helping pre-service teachers understand children's thinking in mathematics and to plan instruction accordingly.

Table 1. Course Point Distribution for the Mathematics Content Course

Integrative seminar	25%
Class preparation and participation	15%
Homework and writing assignment activities from the Beckmann textbook and activities manual	10%
Two eighty-minute exams, 15% each	30%
Final exam	20%
Total	100%

Skills developed include listening to gauge children's understanding, posing questions that uncover children's misconceptions or confusions or that lead children to clarify their thinking, facility in using multiple representations and in drawing correspondences between them, designing and implementing mathematics lessons that are appropriate for elementary age children, and practicing mathematical discourse.

In the course students are also introduced to current practices, changes, and controversy in the field of mathematics education such as the Curriculum, Professional and Assessment Standards of the National Council of Teachers of Mathematics (NCTM) [14], the Massachusetts curriculum standards (i.e., the Massachusetts State Frameworks), and the Massachusetts Comprehensive Assessment System tests, which are the No Child Left Behind (NCLB), high-stakes tests that are used in Massachusetts.

Course readings are drawn from John Van De Walle's *Elementary and Middle School Mathematics: Teaching Developmentally* [21] and the *Young Mathematicians at Work* series [12], by Catherine Twomey Fosnot and Maarten Dolk. De Walle's textbook is chosen because it is a basic, constructivist, standards-based textbook which helps students understand the mathematics content of elementary classrooms. We use the Fosnot and Dolk series because it focuses on how children construct their understanding of the big ideas of mathematics, develop a repertoire of strategies for computation, and "mathematize" their world, and how teachers use rich and complex contexts for this work. Fosnot and Dolk favor Martin Simon's model of a "hypothetical learning trajectory" [17], rather than the traditional, linear sequential model of mathematical learning. We believe that this model of learning helps our students to better understand the mathematics content for themselves, and better understand the complexities of children's mathematical conceptions.

The mathematics methods course included the following group projects: an exploration of NCTM Principles and Standards, a poster on a number system invented by each group, a mathematics and literature set of lessons, and a mathematics game.

In the NCTM Principles and Standards exploration assignment, students use the Web to research the NCTM standards for both Content and Process

Standards. They are asked to pick a mathematical content area and grade level that interests them, and to link from the NCTM standards website to online classroom resources on their topic, including videotapes of classroom instruction, games, and classroom activities. There are multiple goals for this activity: to familiarize students with the NCTM website, to expose students to the kinds of mathematical activities that they will see in Standards-based classrooms, to help students to focus on the Process Standards as a background for their later work in designing their own units and games, and to give them experience with the use of technology for their own learning as adults.

The invented number systems poster has two major goals. One is to help our students understand the base ten system, often for the first time. The other is to give pre-service teachers an experience that in some ways simulates the experience of elementary school children in learning the base ten number system. Understanding the positional decimal system is crucial for understanding key arithmetic algorithms such as add and regroup and subtract and regroup. Students are initially introduced to a number system known to them only as Starfish. They are told a story which has embedded in it a set of guidelines for inventing a number system which uses only five symbols, has exact ways to write any number no matter how large, has place value, works for all four operations, and can show numbers less than a whole. Through a series of explorations they have the opportunity to develop a place value notation for the Starfish number system that is base five.

The purpose of having students create a mathematics game is to make students aware of the potential of games for motivating children and helping them to learn mathematical concepts. While the mathematics methods course introduces students to several games which reinforce number facts or develop computational fluency, the point of this assignment is for students to build children's conceptual understanding. The concepts may be drawn from any of the topics of elementary school mathematics, e.g., numbers and operations, geometry, measurement, data analysis, probability. Students are required to supply commentary with their game which explains how the game is conceptual, not just procedural, and how they would incorporate the game into an overall classroom unit.

The goal of the mathematics and literature assignment is to give students practice in developing a mathematics topic over several days. It also is intended to make students mindful of the Mathematics Curriculum Frameworks and the NCTM Process Standards, and explicitly incorporate those standards into their lessons. Another goal is to give students practice in creating differentiated lessons, i.e., lessons that include extensions and remediation which enrich and attend to the needs of individual learners. This assignment helps students focus on weaving together conceptual and procedural learning for children and designing lessons with a constructivist lens.

Table 2. Course Point Distribution for the Mathematics Methods Course

Integrative seminar	25%
Class preparation and participation	15%
NCTM Principles and Standards Exploration	10%
Starfish (base five) group project and poster	15%
Creation of a mathematics game	15%
Set of mathematics and literature lesson plans	20%
Total	100%

The course point distribution for the mathematics teaching methods course appears in Table 2.

3.4. Structure of the Integrative Seminar

Earlier in this article we described the pivotal role which the integrative seminar plays in bridging theory and practice. We noted that we use a variety of bridging practices including videos of classroom instruction, pieces of actual student work, and elementary school curricula, e.g., *Everyday Math*, from the University of Chicago School Mathematics Project; TERC's *Investigations in Number, Data, and Space*; and *Think Math!*, developed by the Education Development Center in Newton, Massachusetts. For resources we draw from Annenberg Media videos, video footage from TERC's *Relearning to Teach Arithmetic* [15], samples of actual student work, and samples of student assignments from current elementary school mathematics curricula. In addition, we make extensive use of questions and open responses from high-stakes state tests.

Now we look at the required deliverables for the integrative seminar and our grading policies. There are five deliverables in the integrative seminar: reflection paper, weekly journal, portfolio, classroom observation sheets, and clinical interview.

To set the stage for our students to begin thinking about how children build their understanding of mathematics, at the beginning of the semester we ask our students to write a reflection paper on themselves as learners of mathematics. The point of this assignment is for students to identify their attitudes towards mathematics and the experiences that led to these, identify their current strengths and weaknesses, and analyze, looking forward, their needs for areas of growth. Knowing themselves as learners of mathematics can help students understand children as learners of mathematics.

Then, to promote reflection throughout the semester, we require students to keep a journal. The impetus for their reflections may come from articles they are reading, a longstanding question they have had about mathematics or one that arose from a recent class discussion, a conceptual breakthrough and

how that happened, etc. Recording their emerging insights, as they revisit elementary school mathematics, offers students the opportunity to clarify misconceptions and to solidify new understandings.

Finally, to help students gather and reflect upon all their experiences in the learning community and to articulate how their knowledge, beliefs, and understandings in mathematics have formed and changed throughout the learning community, we require students to select items for a portfolio and to assemble them as their final project for the learning community. The portfolio is retrospective, including samples of a student's work which they have selected to demonstrate their growth and development over the semester, as well as prospective; students are required to define areas where they want to grow and where they believe they need more work.

Two other deliverables for the integrative seminar are classroom observation sheets and a clinical interview. To give students opportunities to work with whole-class mathematics instruction and to see current teaching practice, we require them to carry out twice weekly classroom observations. With assistance from the education department, all students receive field placements. They observe a single elementary school classroom throughout most of the semester, seeing at least two mathematics classes per week. This requirement meets state licensure requirements for pre-practicum course hours.

The clinical interview is an assignment where students observe an individual child to determine the child's mathematical understanding and mathematical misconceptions. It gives students the opportunity to enact important teaching skills such as listening and questioning to discern what procedures a child can do mathematically, what the child understands mathematically, and where these do and do not intersect. Students then learn how to use these data to plan for appropriate instruction.

The instructors determine the final grade for the integrative seminar together. The course point distribution for the integrative seminar appears in Table 3.

3.5. Role of Weekly Planning Sessions

We have described the design of our learning community and the structure of its three components: the mathematics content course, the mathematics

Table 3. Course Point Distribution for the Integrative Seminar

Class preparation and participation	50%
Reflection paper on yourself as a mathematics learner	5%
Journal writing	15%
Classroom observation sheets	10%
Clinical interview	10%
Portfolio	10%
Total	100%

methods, and the integrative seminar. What remains to describe is how the learning community design is carried out.

For the mathematics content and pedagogy components, the instructors had their traditional course syllabi as a starting point; however, these syllabi needed to be modified (e.g., in scope and sequence of topics) to align with and support each other. The integrative seminar component, on the other hand, had to be developed from the ground up. Detailed weekly planning and ongoing communication between instructors is necessary to ensure that all three components coordinate to accomplish the goals of the learning community. Here are three of the most important roles of weekly planning sessions.

The first important role of the weekly planning session is to determine the content and format for the weekly integrative seminar. Relevant questions include: Which mathematics content topic(s) shall receive focus? What model of co-teaching shall be used and why? If alternate teaching is used, which instructor will serve as leader? What activities will the supporting instructor engage in and what responses will the lead instructor expect? What teaching method(s) can students expect to observe or practice that will help them teach this mathematics content effectively to children? For each activity, how will students work, individually or in groups? How will the classroom be physically arranged? In what order will the activities be carried out and how much time should be allotted for each? What student pitfalls or confusions can be expected? What strategies can the faculty use to lay bare student confusions, and what strategies can be deployed to help students understand and resolve them? When/how shall student learning be assessed? Who writes up the assignment and sets the deadline for the assignment? How can each instructor explicitly model behaviors, attitudes, and practices of successful learners of mathematics?

The second role of the weekly planning session is for the instructors to coordinate all three components to ensure consistency. Here is one example of how the components were coordinated. We had our students view the Annenberg video, *Valentine Exchange* [20], during an integrative seminar, to observe how fourth grade students represented and solved the problem and to study the classroom teacher's moves. We foreshadowed this seminar twice. The first time we foreshadowed it was at the first meeting of the seminar. Students were asked to solve a version of the classic *Handshake Problem* as a way to get to know each other while also being engaged in mathematics. In this version they had a conversation with each classmate to find something in common; then they had to solve the problem of "how many conversations took place in our classroom."

The second time we foreshadowed the Valentine Exchange problem was at the first meeting of the mathematics content course. Students learned Pólya's method and used it to solve the *Clinking Glasses Problem* of "how many clinks took place." Therefore, students approached the integrative

seminar on the *Valentine Exchange Problem* with the *Handshake Problem* and *Clinking Glasses Problem* as their background.

Subsequent sessions were coordinated with this integrative seminar as well. For example, to focus on teaching, we asked our students to pick one representation that was being created by a single student (or a pair of students) in the *Valentine Exchange* video [20] that they thought would benefit the whole class. They were then asked to explain how they would choose to have that student share, and also to identify what questions they might ask that would help everyone learn something from that sharing. To focus on mathematics, students were asked to compare the *Handshake Problem*, the *Clinking Glasses Problem*, and the *Valentine Exchange Problem* and to explain which of the problems are equivalent. They were also required to explain how the problems differed.

The instructors regularly checked with each other on the progress of individual students. This was important to their weekly planning sessions. They planned and implemented interventions as needed, such as assigning students in particular pairs in order to take care of individual learning needs.

4. SPECIAL CONTRIBUTIONS FROM PARTICIPATION IN INSTITUTES OF THE CENTER FOR PROFICIENCY IN TEACHING MATHEMATICS

In spring 2004, before we conceived of the learning community, one of the authors received notice of a June 2004 institute, Developing Teachers' Mathematical Knowledge for Teaching, to be sponsored by the CPTM at the University of Michigan-Ann Arbor. Seizing on this opportunity to help us strengthen elementary school teacher preparation, we proposed to work together at the institute to design a coherent plan to coordinate our mathematics content and methods courses. Happily, our proposal was accepted. The 2004 CPTM institute, Institute I, and a follow-up program, Institute II, held in Irvine, CA in January 2007, have contributed greatly to our professional development and to shaping the learning community at Simmons College.

First, as participants in CPTM institutes I and II, we joined a diverse community of 70 educators, including mathematics and education faculty who train teachers, mathematics curriculum specialists who work with practicing teachers, and graduate students pursuing research in mathematics education. The leader of the institutes was Deborah Loewenberg Ball, Professor of Mathematics Education and Teacher Education and now Dean of the School of Education at the University of Michigan-Ann Arbor. Since 1997, Deborah Ball and Hyman Bass, Professor of Mathematics at the University of Michigan, have been collaborating on the ". . . mathematical knowledge and resources entailed in the teaching of mathematics at the elementary level" [3].

Ball and Bass are among the foremost leaders and researchers in the scholarship of the preparation of teachers for teaching mathematics. They have earned national recognition for their work; in spring 2007 Ball was elected to the National Academy of Education, and Bass received a 2006 presidential National Medal of Science. Joining the CPTM community provided us a national platform for both sharing and gathering best ideas and practices in teacher development.

Second, what we learned, and continue to learn, from the University of Michigan CPTM institutes is a particular approach to the mathematical preparation of teachers called Mathematical Knowledge for Teaching [5]. Mathematical Knowledge for Teaching is neither pure content knowledge nor pure pedagogical knowledge. Instead, it is a practice-based approach which asks, "What are the mathematical tasks teachers have to do in the course of their work?," as the starting point for determining what knowledge and skills teachers must have in order to carry out those tasks [5]. One example, commonly cited by Ball and her colleagues, of a task in teaching mathematics, is "choosing and using definitions" [5].

Other examples include "analyzing student errors," "giving and evaluating explanations," and, "choosing and using representations and examples" [5]. Attending the CPTM institutes gave us a common framework, Mathematical Knowledge for Teaching, for developing our learning community. We derived individual contributions as well. The mathematics instructor was introduced to the education instructor's network of mathematics educators and to current research in mathematics education; and the education instructor had the opportunity to meet with mathematicians and education colleagues from other institutions who teach pre-service teacher education.

5. BENEFITS OF THE LEARNING COMMUNITY COLLABORATION

5.1. Faculty Perspective

At the outset of this article, we noted that at many institutions, if not most, the responsibility for preparing teachers for teaching mathematics belongs to the mathematics and education departments, but that they work in isolation. By contrast, developing a learning community has been a rich collaboration on multiple levels.

First, we have applied our abilities, experience, and ideas to help address the national problem of teacher preparation for teaching mathematics. Second, we have contributed to our institution's learning community initiative by proposing, planning, and implementing a learning community for prospective teachers. Third, and closely tied to the latter, we have shared in the rare experience, in academe, of team-teaching. Fourth, we have shared in

the professional development experience of learning one of the most cutting-edge approaches to teacher preparation, Mathematical Knowledge for Teaching.

Beyond these important benefits, we have benefited from the intellectual stimulation that comes from working closely with a colleague. This includes crossing disciplinary boundaries and listening to a colleague's beliefs and pedagogical ideas from a different perspective. We were required to think through and properly explain to each other our rationale for the pedagogical decisions we make.

We have garnered individual benefits, too. For the mathematics instructor, the learning community collaboration has provided indirect access to the elementary school classroom, knowledge of current elementary school mathematics curricula and teaching practices, and insight into what it means for activities to be Standards-based. It has also made her aware of the Massachusetts licensure exams for teachers, i.e., the Massachusetts Tests for Educator Licensure, as well as the Massachusetts Comprehensive Assessment System, which is the state's high-stakes, NCLB tests for students in public schools, and of licensure changes that are pending.

For the education instructor, the learning community collaboration has given her knowledge of the topics that are in the mathematics department's content course for elementary school teachers. It has provided her opportunities to see how certain elementary school mathematics topics, e.g., integers and exponents, integers and the distributive property, play out in more advanced mathematics, and how the elementary beginnings fit into this context. She has increased her repertoire of mathematical models, and gained fresh appreciation for how flexibly and efficiently children use, and sometimes create, representations which differ from more traditional ones used by mathematicians. She has had opportunities to delve more deeply into the mathematical foundations of elementary school mathematics and more fully appreciates that the more an elementary school teacher understands the mathematics, the better decisions she can make with her students.

5.2. Student Perspective

To measure the learning community's effectiveness in achieving its stated learning goals of increasing student confidence and professional competence in elementary school mathematics, we have used pre- and post surveys for the mathematics content course and pre-and post-questionnaires for the learning community. For each question, students are asked to scale their answers from strongly disagree to strongly agree along a visual analog scale.

Overall, these surveys indicate that students in the learning community have benefited in several ways. They have strengthened their abilities in problem-solving and critical reading, they have improved their attitudes

Table 4. Changes in Student Attitude Between the Pre- and Post-Surveys for the Mathematics Content Course. Pre-test: Student Responses to Survey at the Beginning of the Semester; Post-test: Student Responses at the End of the Semester. Spring 2007

Question	More favorable	Less favorable	No change
I love to work on math problems.	67%	11%	22%
I get nervous if I don't see the solution quickly.	67%	33%	0%
I'm comfortable reading a math book to find information I want.	78%	11%	11%
I get flustered when I get the wrong answer.	56%	33%	11%
I learn math easily.	67%	11%	22%
When I get stuck, I try to figure things out for myself.	56%	22%	22%

Table 5. Changes in Student Attitude Between the Pre- and Post-Questionnaires for the Learning Community. Pre-test: Student Responses to Survey at the Beginning of the Semester; Post-test: Student Responses at the End of the Semester. Spring 2007

Question	More favorable	Less favorable	No change
I'm confident about explaining how I got my answer to a math problem.	78%	0%	22%
In math class, I often volunteer to show my work.	89%	0%	11%
I find it difficult to come up with more than one way to solve a problem	89%	0%	11%

toward and increased their confidence in mathematics, and they have strengthened their ability to learn mathematics independently. Table 4 summarizes the changes in student responses between the pre- and post-surveys for the mathematics content course, and Table 5 summarizes the changes in student responses between the pre- and post-questionnaires for the learning community.

6. LESSONS LEARNED

Many positive outcomes have emerged from our learning community collaboration. Most encouraging, the assessment data give reason to believe that that the mathematics learning community has helped our pre-service elementary school teachers to gain confidence as well as competence for teaching elementary school mathematics. Also, the learning community has helped the faculty to overcome their sense of isolation. In conceiving and carrying out

this curricular initiative, we have shared a rich experience in professional growth and development that has enabled us to cross disciplinary boundaries and to see the world from each other's perspectives.

It was particularly gratifying to us that external reviewers, who visited the education department in spring 2007, pointed to the mathematics learning community initiative as a model for collaboration between the education department and other liberal arts departments. Along with positive outcomes, we realize that our mathematics learning community is a work in progress. We acknowledge several important lessons learned.

In the area of administration, we have identified three lessons. First, we have learned that coordinating and planning require adequate course release time. This, in turn, requires both administrative and departmental buy-in. Since release time for each instructor was only available in year one, coordination was less smooth in year two.

Second, regarding logistics, we learned that it is crucial to communicate with the Registrar's office to ensure that registration for the learning community goes smoothly. Third, we learned that we need to keep all faculty who advise pre-service elementary school teachers apprised of registration specifics so that they can help advisees develop four-year plans which allow them to take the mathematics content and teaching methods courses in the same semester and year.

In the area of curriculum, there have also been lessons learned. First, because pre-service elementary school teachers are only required to take one mathematics course, they are limited in their opportunities to learn elementary school mathematics. Due to time constraints, the chief focus is number systems and operations. However, starting in 2009, Massachusetts will require that prospective teachers take a separate content knowledge test in mathematics for state licensure. As the state requirements for elementary school mathematics content increase, we will need to respond to that change.

Second, state licensure requirements are an important consideration in all teacher preparation programs. In year two, we felt increased pressure to spend time preparing our students for state licensure exams; therefore, students had less time in year two, than in year one, for exploring the connections between the mathematics topics and their treatment in the undergraduate mathematics content course and the way those topics are developed in elementary school mathematics curricula.

Third, in the area of mathematical content, we found that in both years one and two, a specific student weakness has been exponents. This weakness is significant because it is an impediment to learning number base systems. Fourth, in the area of teaching methods, we have learned that children's stories, which author Apostolos Doxiadis asserts ". . . ease children's transition to abstract mathematics" [11], also motivate pre-service teachers and help them to solidify understanding of mathematical concepts. In the learning community we used a small set of mathematically rich stories to motivate

and to build emotional and intellectual connections. These stories allowed us to evoke particular mathematical themes quickly and powerfully.

Last, but perhaps most important, we learned lessons about co-teaching. For faculty to engage in collaborative teaching requires that they be open—open to learning each other's content, negotiating class time, and sharing the syllabus. Also, learning to team-teach takes time, and we did get institutional support for reduced load in the first year. In year one, we were getting to know each other as well as learning each other's discipline and teaching practices; we mostly used the alternate teaching model. A year two improvement was that we were more comfortable and confident and began to team-teach. Particularly satisfying, we had the pleasure of being able to share our mutual enthusiasm for teaching, observe a colleague's knowledge and passion for her subject, and improve our own teaching by watching the other. This has made classes productive and meaningful.

7. FUTURE DIRECTIONS

In the first two years of our collaboration, we have been immersed in developing and implementing an interdisciplinary learning community. Based on our experiences and lessons learned, here are the challenges we plan to address as we move forward.

There is the continuing need to improve the process of integrating the mathematics content and methods course. For example, we have felt some uncertainty, sensed by students, about what is the appropriate level and kind of homework for the integrative seminar. We will revise old assignments and design new ones with an eye to more effectively bridging content and pedagogy.

Also, we need to capitalize on the potential efficiency offered by teaching content and methods together. At present, 80 percent of the mathematics concepts and examples that are discussed in the mathematics methods course come from number systems and operations. Since those concepts are presented in the mathematics content course, less time needs to be spent reviewing them in the methods course. Further, we would like to increase the quantity and quality of our team-teaching and we are eager to learn and more fully understand the practice-based approach to teaching.

In another direction, we need to consider possible curricular additions. Pending changes on elementary school teacher licensure exams in Massachusetts will more than double the number of questions in mathematics; and, pre-service teachers will be required to pass the mathematics subject area test. To meet those changes, the education department at Simmons has asked the mathematics department to develop a new mathematics course that extends the existing mathematics course. This will give students expanded opportunities for learning the content of elementary school mathematics.

We anticipate that these challenges will animate and direct our work in the year ahead.

REFERENCES

1. Ball, D. L. 2000. Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education.* 51(3): 241–247.
2. Ball, D. L., H. C. Hill, and H. Bass. 2005. Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator.* (Fall): 14–46. March 17, 2008. http://www.aft.org/pubs-reports/american_educator/issues/fall2005/BallF05.pdf (Accessed September 28, 2009).
3. Ball, D. L., H. Bass, T. Boerst, et al. January 26, 2007. *Learning Mathematics for Teaching Project.* University of Michigan, Ann Arbor. http://sitemaker.umich.edu/lmt/people (Accessed September 28, 2009).
4. Ball, D. L. 2003. Mathematical Proficiency for All Students: Toward a Strategic Research and Development Program in Mathematics Education. Rand Report prepared for the Office of Educational Research and Improvement, U.S. Department of Education. Chapter Two Teachers' Mathematical Knowledge: Its Development and Use in Teaching. RAND Education, Inc. March 17, 2008. https://rand.org/pubs/monograph_reports/MR1643/MR1643.ch2.pdf (Accessed September 28, 2009).
5. Ball, D. L. May 2003. Using Content Knowledge in Teaching: What Do Teachers Have to Do, and Therefore Have to Learn? Address. The Third Annual Conference on Sustainability of Systemic Reform. TERC, Cambridge, MA. March 17, 2008. http://sustainability2003.terc.edu/go.cfm/key_1 (Accessed September 28, 2009).
6. Beckmann, S. 2005. *Mathematics for elementary teachers.* Boston: Pearson Education, Inc.
7. Beckmann, S. 2005. *Class activities to accompany mathematics for elementary teachers.* Boston: Pearson Education, Inc.
8. Collaborative Teaching Models and Templates. Multidisciplinary Learning Team Teaching Initiative. University of Michigan Office of the Provost. March 17, 2008. http://www.provost.umich.edu/programs/MLTT/resources/collabteaching.pdf (Accessed September 28, 2009).
9. Conference Board of the Mathematical Sciences. 2001. *The mathematical education of teachers.* Providence, RI, Washington, DC: American Mathematical Society and Mathematical Association of America.
10. Devlin, K. September 2007. What Is Conceptual Understanding? Devlin's Angle. MAA Online. The Mathematical Association of America. March 17, 2008. http://www.maa.org/devlin/devlin_09_07.html (Accessed September 28, 2009).

11. Doxiadis, A. 3 January 2003. Embedding Mathematics in the Soul: Narrative as a Force in Mathematics Education. Address. The Third Mediterranean Conference of Mathematics Education. Athens, Greece. March 17, 2008. http://www.apostolosdoxiadis.com/files/essays/embeddingmath.pdf (Accessed September 28, 2009).
12. Fosnot, C. T., and M. Dolk. 2002. *Young mathematicians at work.* Portsmouth, NH: Heinemann.
13. Ginsburg, A., and G. Cooke, S. Leinwand, et al. November 2005. Reassessing U.S. International Mathematics Performance: New Findings from the 2003 TIMSS and PISA. American Institutes for Research. March 17, 2008. http://www.air.org/news/documents/TIMSS_PISA20math20study.pdf (Accessed September 28, 2009).
14. NCTM. The National Council of Teachers of Mathematics. *Principles and standards for school mathematics.* 2000. Reston, VA: NCTM.
15. Russell, S. Jo. 1999. Relearning to teach arithmetic. Cambridge, MA: TERC. March 17, 2008. http://www.terc.edu/work/610.html (Accessed September 28, 2009).
16. Science and Engineering Indicators 2006. February 23, 2006. The National Science Board. March 17, 2008. http://www.nsf.gov/statistics/seind06/ (Accessed September 28, 2009).
17. Simon, M. A. 1995. Elaborating models of mathematics teaching: A response to Steffe and D'Ambrosio. *Journal for Research in Mathematics Education.* 26(2): 160–162.
18. Thames, M. H. 2006. Using Math to Teach Math: Mathematicians and Educators Investigate the Mathematics Needed for Teaching. Workshop publication. Critical Issues in Mathematics Education Workshop: Mathematical Knowledge for Teaching (K-8): Why, What, and How? Pacific Grove, CA, May 25–28, 2005. The Mathematical Sciences Research Institute. March 17, 2008. http://www.msri.org/calendar/attachments/workshops/318/612msriThames.pdf (Accessed September 28, 2009).
19. NCES. The National Center for Education Statistics. *The Nation's Report Card: 12th-Grade Reading and Mathematics 2005.* Executive Summary. February 22, 2007. March 17, 2008. http://nces.ed.gov/nationsreportcard/pdf/main2005/2007468_1.pdf (Accessed September 28, 2009).
20. Valentine Exchange. Teaching Math: A Video Library, K-4. Video 42. Annenberg Media. March 17, 2008. http://www.learner.org/resources/series32.html (Accessed September 28, 2009).
21. Van De Walle, J. A. 2004. *Elementary and Middle School Mathematics: Teaching Developmentally, 5th* ed. Boston: Pearson Education, Inc.
22. What Are Learning Communities?. Washington Center for Improving Undergraduate Education. March 17, 2008. http://www.evergreen.edu/washcenter/lcfaq.htm#top (Accessed September 28, 2009).

BIOGRAPHICAL SKETCHES

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